

## Exercises for AO2 Trig and log Proofs

1 By putting  $u = \log_4 r$ , show that  $\log_4 r = \frac{1}{2} \log_2 r$

**A**  $u = \log_4 r$  so  $r = 4^u$

$$r = 4^u$$

$$r = 2^{2u}$$

$$r^{\frac{1}{2}} = 2^u$$

$$\log_2 r^{\frac{1}{2}} = u$$

$$\frac{1}{2} \log_2 r = \log_4 r$$

**B**  $\log_4 r = \log_{2^2} r$

$$\log_4 r = \log_2 \sqrt{r}$$

$$\log_4 r = \frac{1}{2} \log_2 r$$

**C**  $u = \log_4 r$  so  $r = 4^u$

$$r = 4^u = 2 \times 2^u$$

$$\frac{r}{2} = 2^u$$

$$\frac{1}{2} \log_2 r = u$$

$$\frac{1}{2} \log_2 r = u$$

**D**  $\log_4 r = \frac{\ln r}{\ln 4}$

$$\frac{\ln r}{\ln 4} = \frac{\ln r}{2 \ln 2}$$

$$\frac{\ln r}{2 \ln 2} = \frac{1}{2} \log_2 r$$

**E**  $\frac{1}{2} \log_2 r = \log_4 r$

$$\frac{\log_2 r}{\log_2 4} = \log_4 r$$

$$\log_4 r = \log_4 r$$

## Exercises for AO2 Trig and log Proofs

2. Prove that  $\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$

**A** 
$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$$

$$(1+\sin 2x)(1-\tan x) = \cos 2x(1+\tan x)$$

$$1 + \sin 2x - \tan x - \sin 2x \tan x = \cos 2x \tan x$$

$$\cos x + \cos x \sin 2x - \sin x - \sin 2x \sin x = \cos 2x \cos x + \cos 2x \sin x$$

$$\cos x + \cos x \sin 2x - \sin x - \cos 2x \sin x = \cos 2x \cos x + \sin 2x \sin x$$

$$\cos x - \sin x = \cos x - \sin x$$

**B** 
$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x} \div \frac{\cos^2 x}{\cos^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\sec^2 x + 2\tan x}{1-\tan^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan^2 x + 2\tan x}{1-\tan^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{(1+\tan x)^2}{(1+\tan x)(1-\tan x)}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$$

**C** 
$$\frac{1+\tan x}{1-\tan x} = \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$$

## Exercises for AO2 Trig and log Proofs

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{1+\sin 2x}{\cos 2x}$$

**D**

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x} \div \frac{\cos x}{\cos x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$$

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